



INSTITUTE OF DISTANCE AND OPEN LEARNING

Gauhati University

HOME ASSIGNMENT

M. A./M.Sc. Mathematics
(4th Semester) 2011-2012 Session

Guidelines for Submission:

1. Write your name, session, roll number, the topic selected and the title of the answer *clearly on the top*.
2. Each of the two topics given in each paper will be answered as **two essays** of not more than 500 words each. There will be negative marking for writing in excess of the word-limit.
3. Each answer (essay) carries a weightage of **10 marks**. (10 marks x 2 essays = 20 marks).
4. Keep a margin of about 1 inch on each side of the page.
5. You can submit the essay written in your own hand-writing on clean, foolscap sheets, or A-4 sized paper.
6. In case you prefer to submit type-written answers, make sure that there are no typing errors which will deduct from the overall impression.
7. Do not submit commercially purchased answers as such a practice is deemed to be unfair.
8. Please submit your assignment by **15th May, 2013**.

401. Graph Theory (answer any two) 2×10=20

1. Introduce the concept of connectivity parameters κ and λ of a graph with suitable examples. Does there always exist a graph with $\kappa = r$, $\lambda = S$ and $\delta = t$ for any three integers r, s, t such that $0 < r \leq s \leq t$? It so establish the result. 10
2. Discuss the concept of planar graphs: starting with Euler's polyhedron formula and giving kuratowski's characterization of planar graphs. 10
3. Introduce the concept of colorability in Graphs. Discuss four color conjecture and prove five color theorem. 10

402. Numerical Analysis (answer any two) 2×10=20

1. Solve 10

$$10x - 7y + 3z + 5u = 6$$

$$-6x + 8y - z - 4u = 5$$

$$3x + y + 4z + 11u = 2$$

$$5x - 9y - 2z + 4u = 7$$
 By Gauss elimination method.
2. Given the values 10

x :	5	7	11	13	17
F(x)	150	392	1452	2366	5202

 Evaluate f(9) using (i) Lagrange's formula
 (ii) Newton's divided difference formula
3. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using 10
 (i) Simpson's 3/8 rule
 (ii) Weddle's rule and compare the results with its actual value.

403. (A) Functional Analysis (optional) (answer any two parts) 10+10=20

1. (a) Prove that every linear operator is bounded in a finite dimensional named linear space. 5
 (b) show that the functionals defined as $C[a, b]$ by $f(x) = \int_a^b x(t)y_0(t)dt, y_0 \in C[a, b]$ is linear and bounded. 5
2. (a) Show that the dual space of ℓ^1 is ℓ^∞ 6
 (b) Show that R^n is a Hilbert space with inner product defined by 4

$$\langle x, y \rangle = \epsilon_1 \eta_1 + \dots + \epsilon_n \eta_n$$
 where $x = (\epsilon_1, \dots, \epsilon_n)$ and $y = (\eta_1, \dots, \eta_n)$
3. State fundamental theorems on Banachspaas. Prove any one of them in two different methods.
 Reference: (1) Functional Analysis with Application by kreyszig.
 (2) Functional Analysis by Megginson.

- (B) Fluid Dynamics (optional)** 5×4=20
1. Discuss the velocity distribution in the flow of a viscous incompressible fluid between two parallel plates taking the fluid properties as constant in the following cases.
 - (i) Plane Couette flow. 5
 - (ii) Generalized plane Couette Flow. 5
 2. Establish the relation between wave velocity and group velocity with emphasize on dynamical significance of group velocity. 5
 3. Write an essay about Prandtl's boundary layer theory with its importance in Fluid Dynamics. 5
- 404. (A) Mathematical Statistics (optional) (answer any two)** 2×10=20
1. State and prove Baye's theorem. 10
Show that if A and B are independent then A' and B' are independent. Also A and B' are independent.
 2. Write short notes on any two of the following 10
 - (a) Binomial Distribution
 - (b) Normal Distribution
 - (c) Regression and correlation
 3. State the properties of *t*-distribution. How does it differ from a standard normal distribution? Mention some applications of *t*-distribution. 10
- (B) Dynamical System and Fractal Geometry (optional)** 10+10=20
1. Construct the Sierpinski triangle and explain why it is self similar. 10
 2. What is box-dimension. Find the box dimensions of Sierpinskiu triangle and Cantor Set. 10
- 405. (A) Fuzzy Sets and Their Applications (optional) (answer any two)** 2×10=20
1. What is the role of α -cuts and strong α -cuts in fuzzy set theory? What are the differences between them? Describe these concepts in your own words. 10
 2. Prove that the properties of symmetry, reflexivity and transitivity are preserved under inversion for both crisp and fuzzy relations. 10
 3. Give examples from daily life of the following types of fuzzy propositions and express the propositions in its canonical form:
 - (i) Unconditional and qualified propositions. 5
 - (ii) Conditional and unqualified propositions. 5
- (B) General Theory of Relativity and Cosmology (optional)** 5×4=20
- 1) State Einstein's principle of equivalence and discuss its application in brief to red shift of spectral times and curvature of light in gravitational field. 5
 - 2) State Einstein's law of gravitation. Modify it for empty space and obtain Schwargschild's solution for an isolated particle continually at rest at the origin. 5
 - 3) Derive Friedman-Robertson-Wlaker (FRW) model and discuss in detail its dynamical consequences. 5
 - 4) Discuss the physical properties of de-sitter universe and compare it with those of the actual universe. 5

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